Coverage and Capacity Analysis of mmWave Cellular Systems

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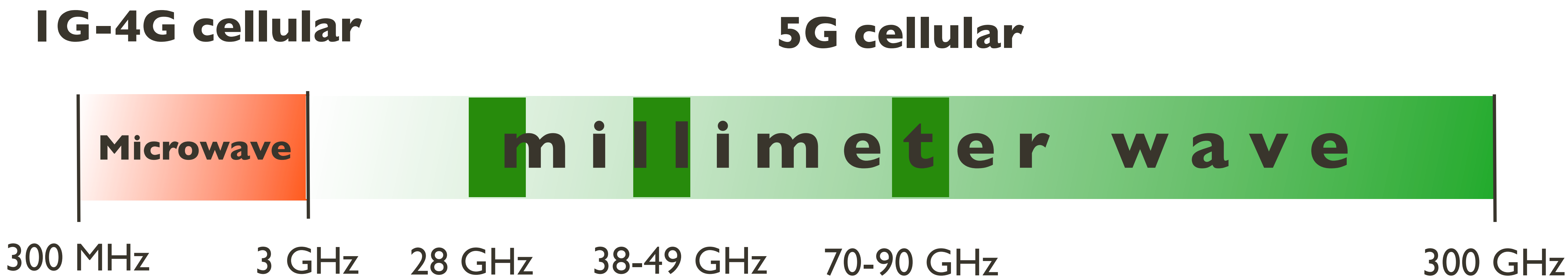
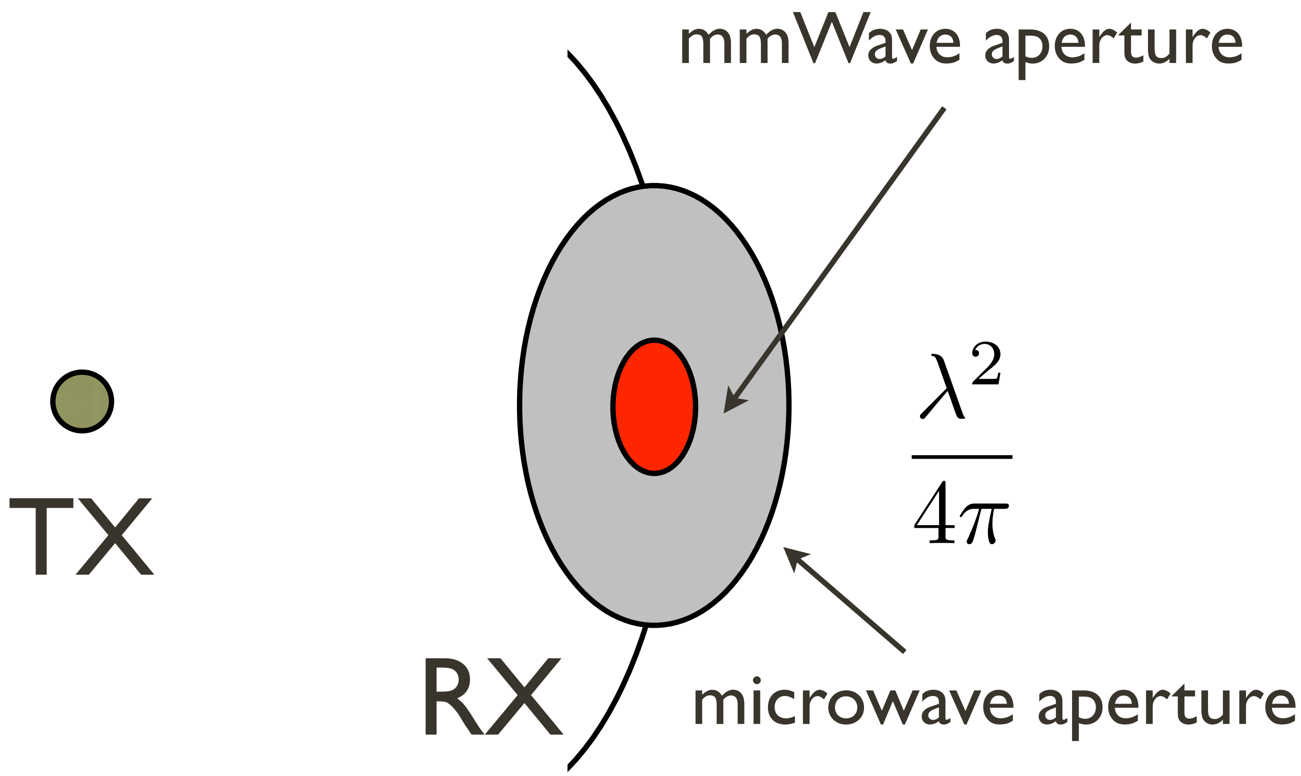
Why mmWave for Cellular?

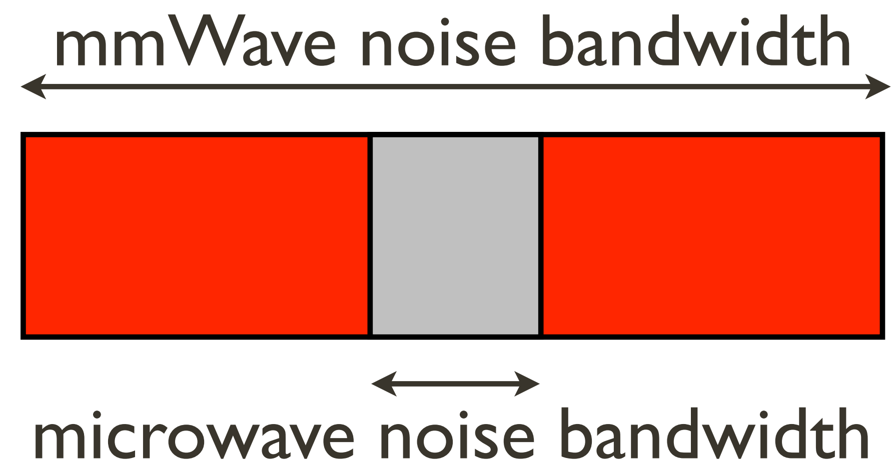


- Huge amount of spectrum available in mmWave bands*
  - Cellular systems live with limited microwave spectrum ~ 600MHz
  - 29GHz **possibly** available in 23GHz, LMDS, 38, 40, 46, 47, 49, and E-band
- Technology advances make mmWave possible
  - Silicon-based technology enables low-cost highly-packed mmWave RFIC**
  - Hybrid precoding solves RF limitations
  - Commercial products already available (or soon) for PAN and LAN

* Z. Pi,, and F. Khan. "An introduction to millimeter-wave mobile broadband systems." IEEE *Communications Magazine,* vol. 49, no. 6, pp.101-107, Jun. 2011.
** T.S. Rappaport, J. N. Murdock, and F. Gutierrez. "State of the art in 60-GHz integrated circuits and systems for wireless communications." *Proceedings of the IEEE*, vol. 99, no. 8, pp:1390-1436, 2011

The Need for Gain

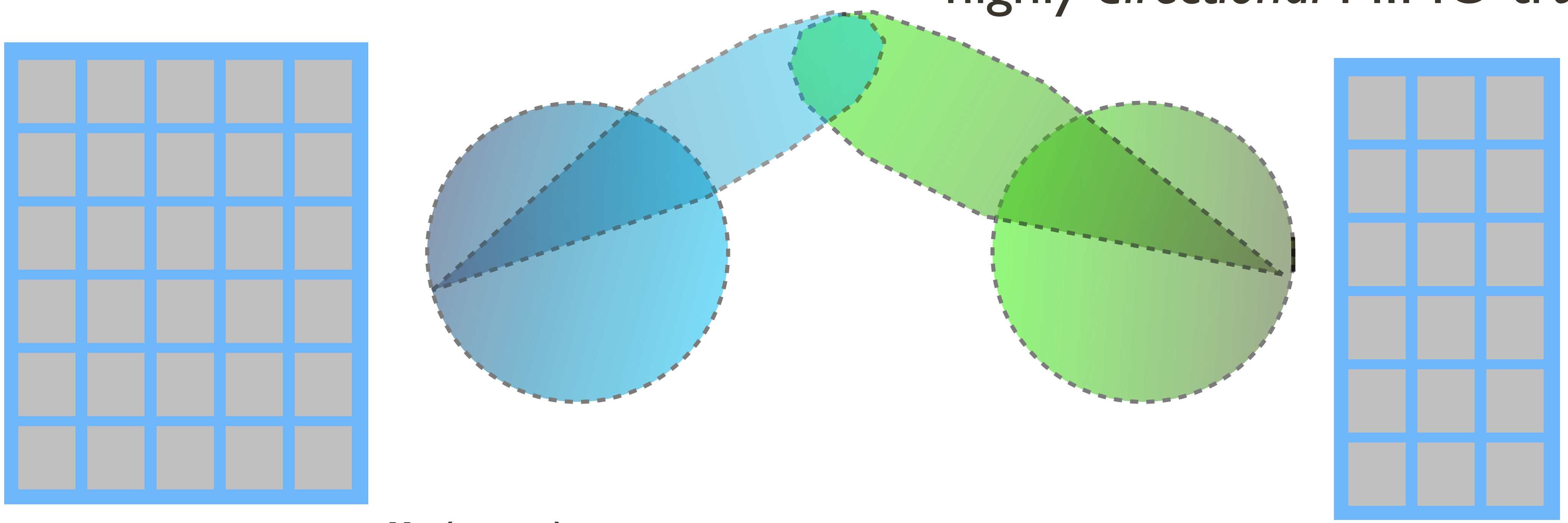




- Smaller wavelength means smaller captured energy at antenna
  - 3GHz->30GHz gives 20dB extra path loss due to aperture
- Larger bandwidth means higher noise power and lower SNR
  - 50MHz -> 500MHz bandwidth gives 10dB extra noise power

Solution: Exploit array gain from large antenna arrays

Antenna Arrays are Important

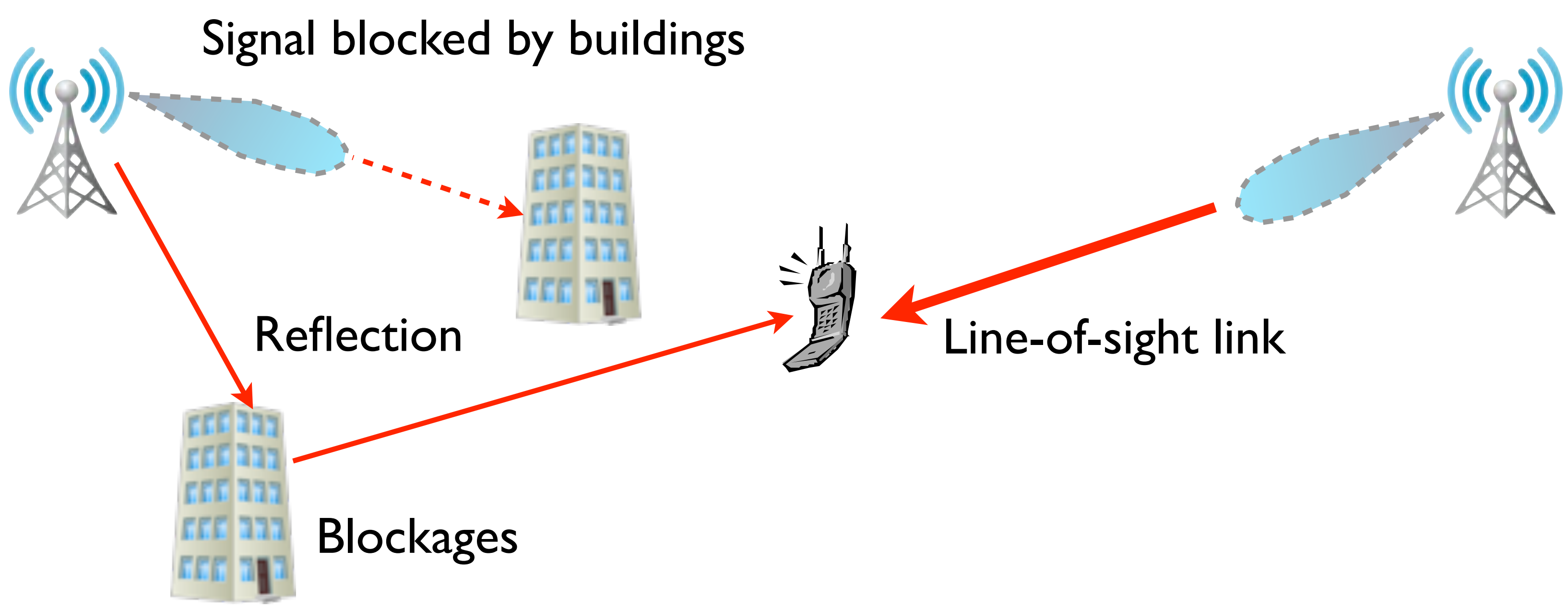


- Narrow beams are a new feature of mmWave
  - Reduces fading, multi-path, and interference
  - Implemented in analog due to hardware constraints

Arrays will change system design principles

O. El Ayach, S. Abu-Surra, S. Rajagopal, Z. Pi, and R. W. Heath, Jr., `` Spatially Sparse Precoding in Millimeter Wave MIMO Systems," submitted to IEEE Trans. on Wireless, May 2013. Available on ArXiv.

Different Propagation Behavior

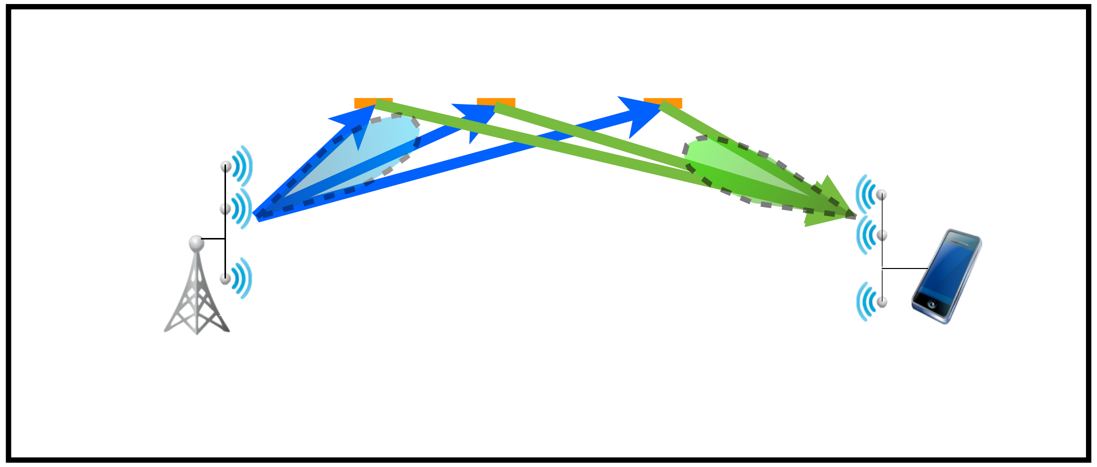


- Buildings block Line-of-sight (LOS) paths
  - mmWave signal suffer from high penetration losses
- Reflections can establish non-LOS links
  - Best non-LOS links still tens of dB weaker than LOS signals
- Different characteristics for LOS & non-LOS
  - Path loss exponent is around 2 for LOS, and 4 for non-LOS

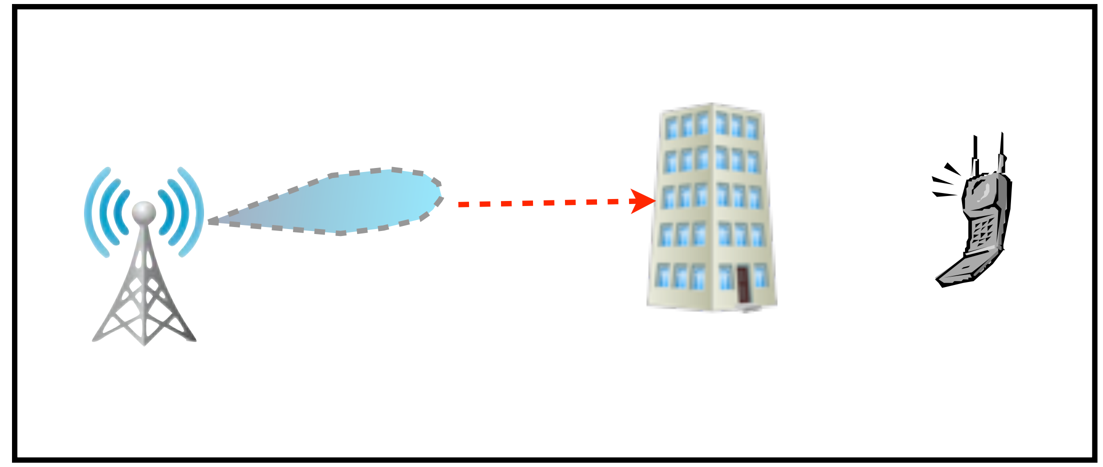
Need to distinguish between LOS and non-LOS paths

T. S. Rappaport, et al. , "Millimeter Wave Mobile Communications for 5G Cellular: It Will Work!." IEEE Access, vol.1, no.1, pp.335-349, 2013

Challenges of mmWave Analysis



Directional Beamforming (BF)



LOS & non-LOS links

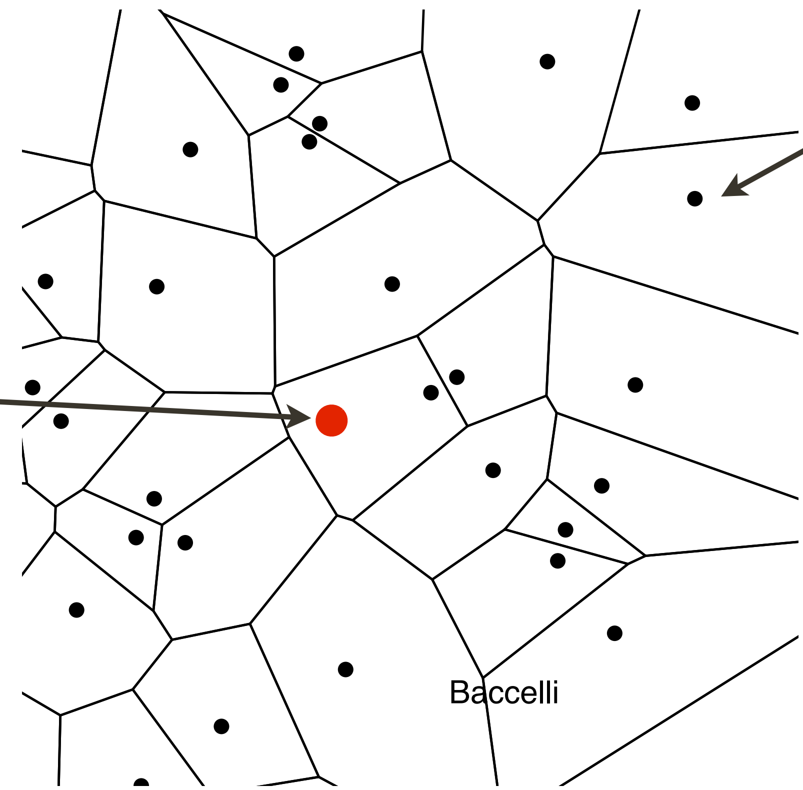
- Need to incorporate directional beamforming
  - RX and TX communicate via main lobes to achieve array gain
  - Steering directions at interfering BSs are random
- Need to distinguish LOS and non-LOS paths
  - Dramatically different characteristics in LOS & non-LOS channels
  - Better characterize blockages

How to including beamforming + blockages in mmWave cellular analysis?

Stochastic Geometry for mmWave Cellular System Analysis

Stochastic Geometry for Cellular

performance analyzed for a typical user



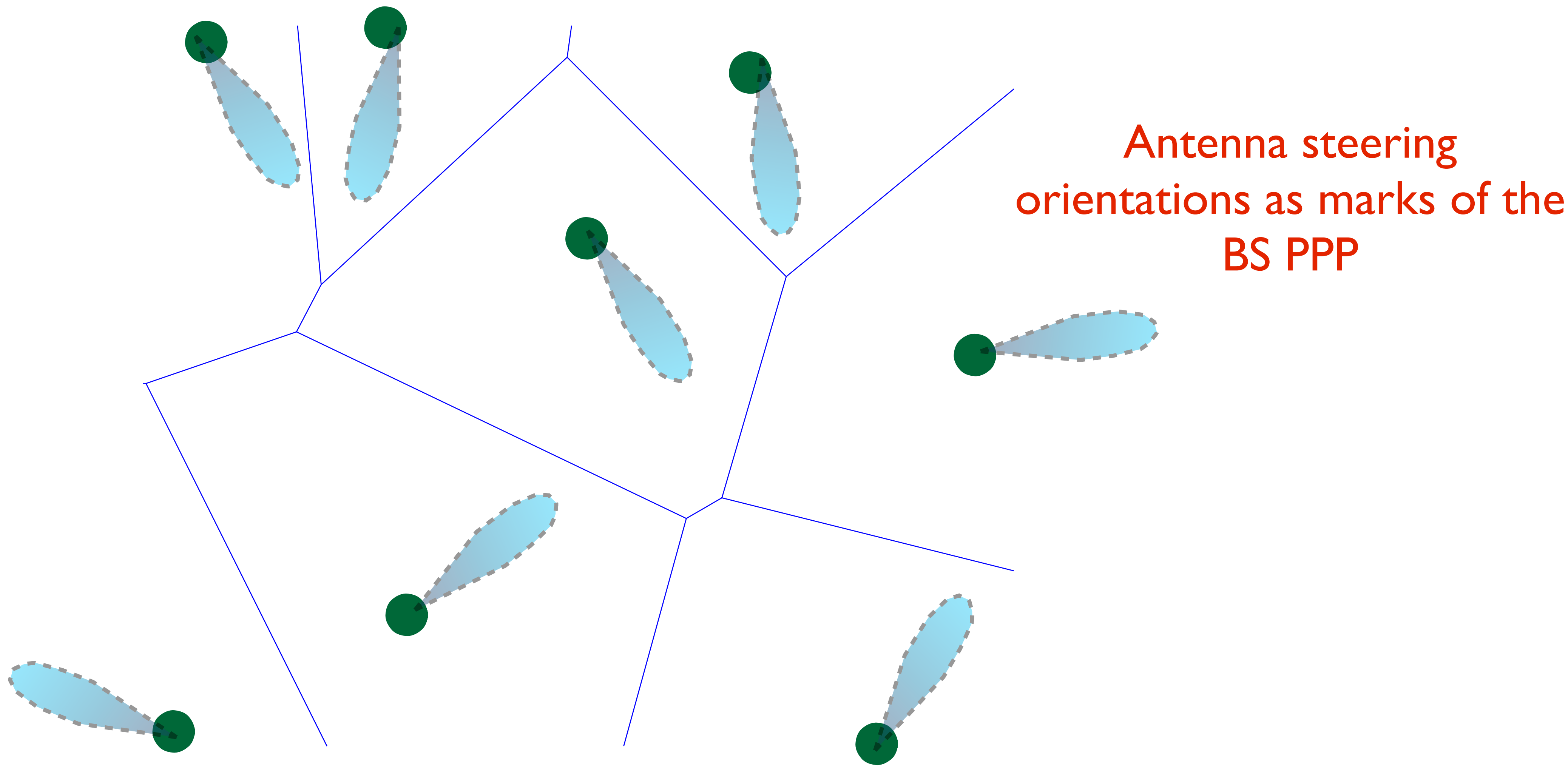
base station locations distributed (usually) as a Poisson point process (PPP)

- Stochastic geometry is a tool for analyzing microwave cellular
  - Reasonable fit with real deployments
  - Closed form solutions for coverage probability available
  - Provides a system-wide performance characterization

Need to incorporate LOS/non-LOS links and directional antennas

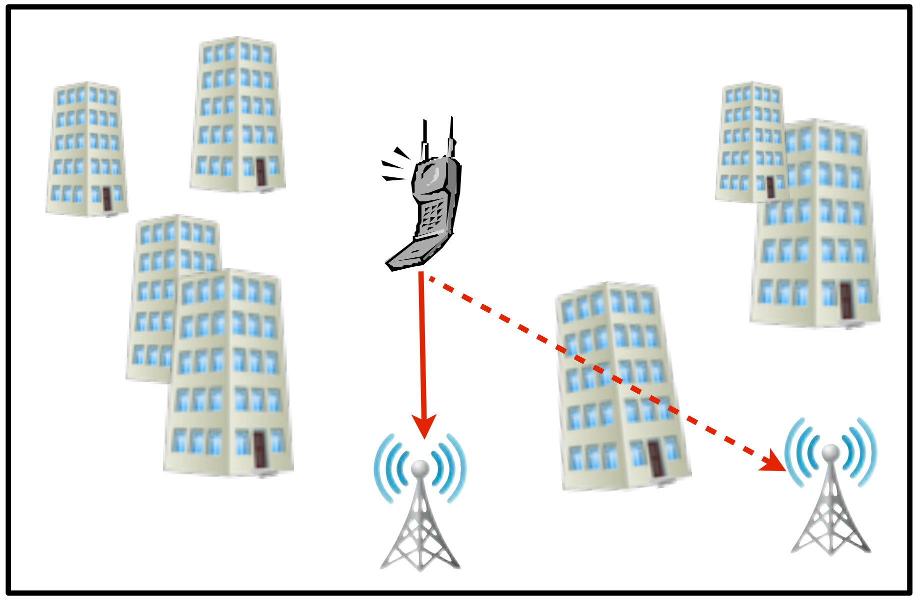
J. G. Andrews, F. Baccelli, and R. K. Ganti, "A Tractable Approach to Coverage and Rate in Cellular Networks", IEEE Transactions on Communications, November 2011.
T. X. Brown, "Cellular performance bounds via shotgun cellular systems," IEEE JSAC, vol.18, no.11, pp.2443,2455, Nov. 2000.

Poisson Point Processes

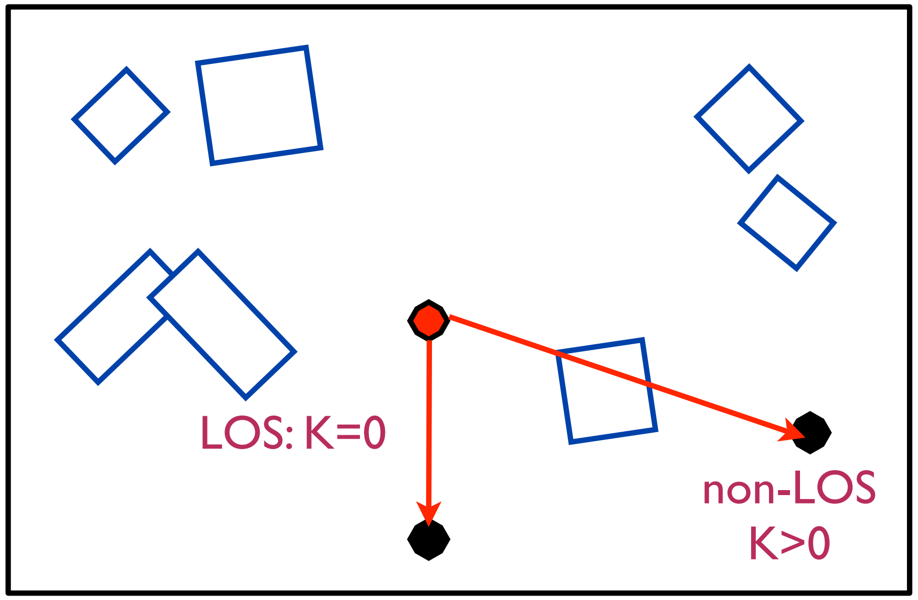


- Poisson point process (PPP): the simplest point process
  - # of points is a Poisson variable with mean λS
  - Given N points in certain area, locations independent
  - Useful results like Campbell's Theorem & Displacement Theorem apply
  - Assigning each point an i.i.d. random variable forms a marked PPP

Blockages in mmWave



Randomly located buildings

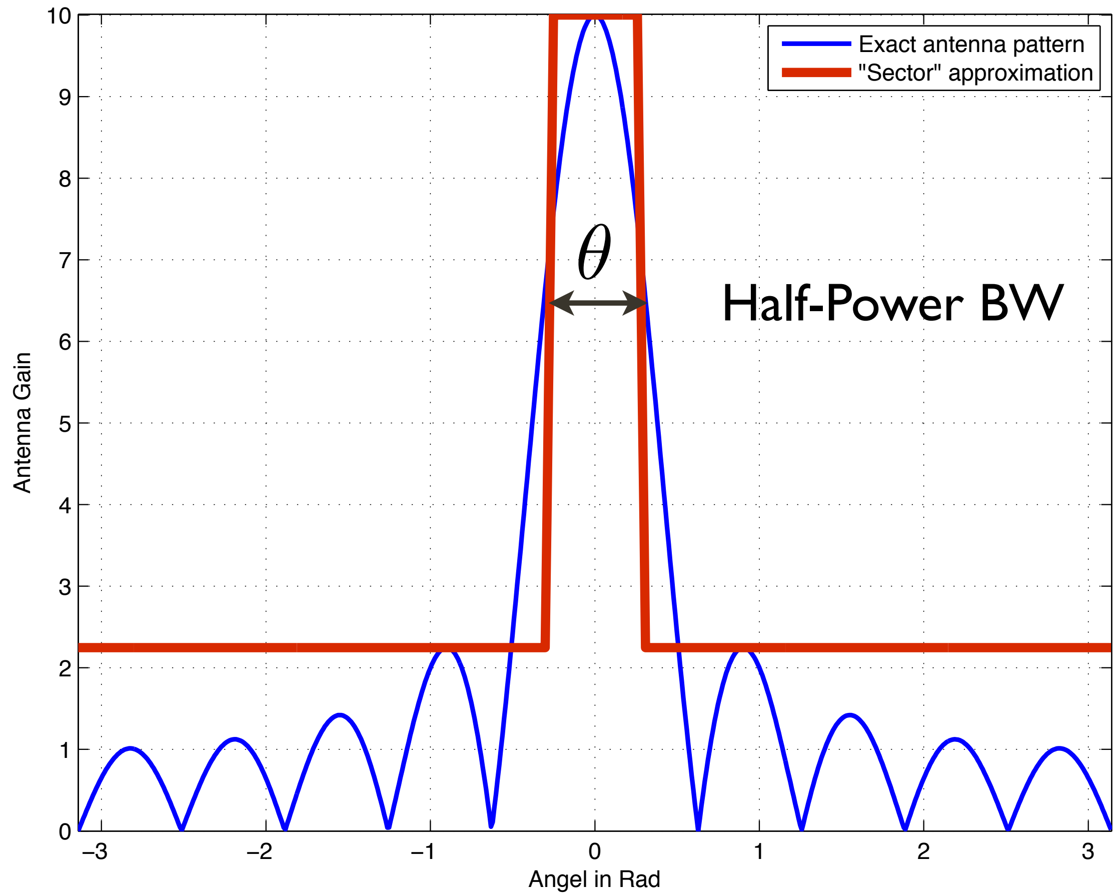


Boolean scheme of rectangles
K: # of blockages on a link

- Use random shape theory to model buildings
  - Model random buildings as rectangle Boolean scheme
  - Buildings distributed as PPP with independent sizes & orientations
- Compute the LOS probability based on the building model
  - # of blockages on a link is a Poisson random variable
  - The LOS probability that no blockage on a link of length R is $e^{-\beta R}$

Tianyang Bai and R. W. Heath, Jr., ``Using Random Shape Theory to Model Blockage in Random Cellular Networks," Proc. of the International Conf. on Signal Processing and Communications, Bangalore, India, July 22-25, 2012.

Directional Transmission at the BS



Sector antenna approximation

Main lobe beamwidth:

$$\theta = 2\arcsin\left(\frac{2.782}{\pi N_t}\right)$$

Main lobe array gain:

$$M = N_t \qquad \text{\# antennas}$$

Front-back ratio:

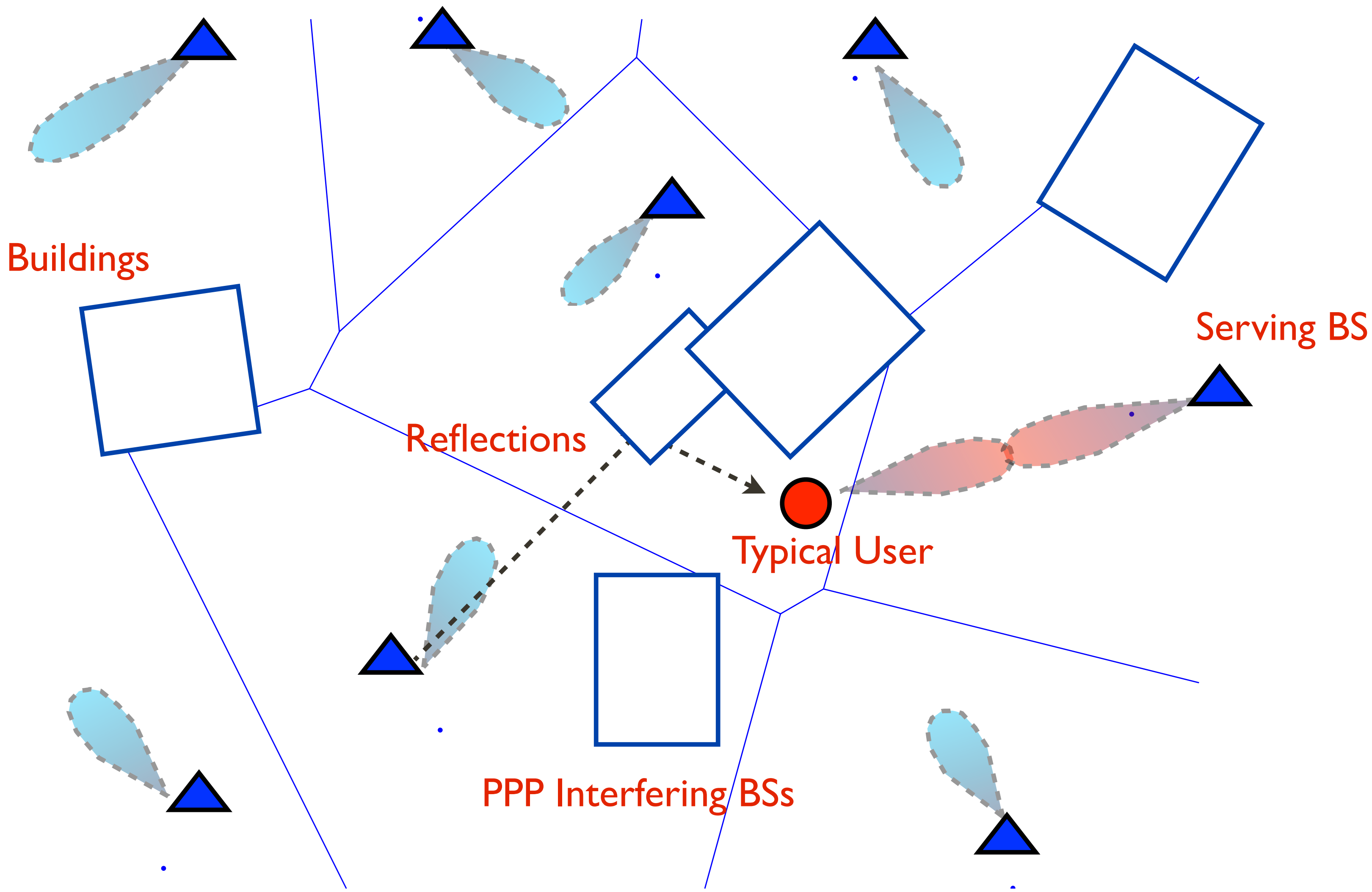
$$\text{FBR} = \sin\left(\frac{3\pi}{2N_t}\right)$$

Back lobe gain:

$$m = N_t \times \text{FBR}$$

- Each base station is marked with a directional antenna
  - Antenna directions of interferers are uniformly distributed
- Use "sector" pattern in analysis for simplicity
  - Equivalent to uniform linear array of N_t antennas with spacing $\lambda/2$

Proposed mmWave Model



- Use stochastic geometry to model BSs as marked PPP
  - Model the steering directions as independent marks of the BSs
- Use random shape theory to model buildings
  - Model the building as rectangle Boolean schemes
  - Different path loss exponents for LOS and non-LOS paths

System Parameters

- Different path loss model for LOS and nonLOS links
  - Line-of-sight with probability $\mathrm{e}^{-\beta R}$
  - LOS path Loss in dB: $\mathrm{PL}_1 = C + 20 \log R(\mathrm{m})$
  - Non-LOS path loss in dB: $\mathrm{PL}_2 = C + K + 40 \log R(\mathrm{m})$
  - 28Ghz system: let C=50 dB, K=10 dB
- General small scale fading h
  - No fading case: small scaling fading is minor in mmWave [RapSun]
- Link budget
  - Tx antenna input power: 30dBm
  - Signal bandwidth: 500 MHz (Noise: -87 dBm)
  - Noise figure: 5dB

Results on Coverage

Coverage Analysis

Serving BS and User connect via main lobe

$$\text{SINR} = \frac{N_{\mathrm{r}} N_{\mathrm{t}} H_0 \ell(r_0)}{N_0/P_{\mathrm{t}} + \sum_{k>0} A_k B_k H_k \ell(r_k)},$$

General small-scale fading

where $H_0\ell(r_0) = \min_{k>0} \{H_k \ell(r_k)\},$ Connecting to the strongest signal before BF

$$A_k = \begin{cases} N_{\mathrm{t}} & \text{w. p. } \frac{\theta_{\mathrm{t}}}{2\pi} \\ N_{\mathrm{t}}\mathrm{FBR}_{\mathrm{t}} & \text{w. p. } 1 - \frac{\theta_{\mathrm{t}}}{2\pi} \end{cases},$$

Array gain of the TX antenna

$$B_k = \begin{cases} N_{\mathrm{r}} & \text{w. p. } \frac{\theta_{\mathrm{r}}}{2\pi} \\ N_{\mathrm{r}}\mathrm{FBR}_{\mathrm{r}} & \text{w. p. } 1 - \frac{\theta_{\mathrm{r}}}{2\pi} \end{cases},$$

Array gain of the RX antenna

$$\ell(x) = \begin{cases} Cx^{-2} & \text{w. p. } \mathrm{e}^{-\beta x} \\ C\ Kx^{-4} & \text{w. p. } 1 - \mathrm{e}^{-\beta x} \end{cases}.$$

Path Loss of LOS or non-LOS

Use stochastic geometry to compute SINR distribution

Coverage Probability of mmWave

Main Theorem [mmWave Coverage probability]

The coverage probability $\mathbb{P}[\mathrm{SINR} > T]$ can be computed as

$$\mathbb{P}(SINR > T) = \int_{-\infty}^{\infty}\int_{0}^{\infty} U(x,t) f_{L^*}(x) \frac{\mathrm{e}^{j2\pi t/T} - 1}{j2\pi t} \mathrm{d}x\mathrm{d}t$$

where

$$U(x,s) = \exp\left[-\frac{sx}{M_\mathrm{r}M_\mathrm{t}\rho} + \int_x^\infty \left(p_\mathrm{t}p_\mathrm{r}\mathrm{e}^{-\frac{sx}{u}} + (1-p_\mathrm{t})p_\mathrm{r}\mathrm{e}^{-\frac{sxm_\mathrm{t}}{uM_\mathrm{t}}} + p_\mathrm{t}(1-p_\mathrm{r})\mathrm{e}^{-\frac{sxm_\mathrm{r}}{uM_\mathrm{r}}} + (1-p_\mathrm{t})(1-p_\mathrm{r})\mathrm{e}^{-\frac{sxm_\mathrm{t}m_\mathrm{r}}{uM_\mathrm{r}M_\mathrm{t}}} - 1\right)\Lambda(\mathrm{d}u)\right],$$

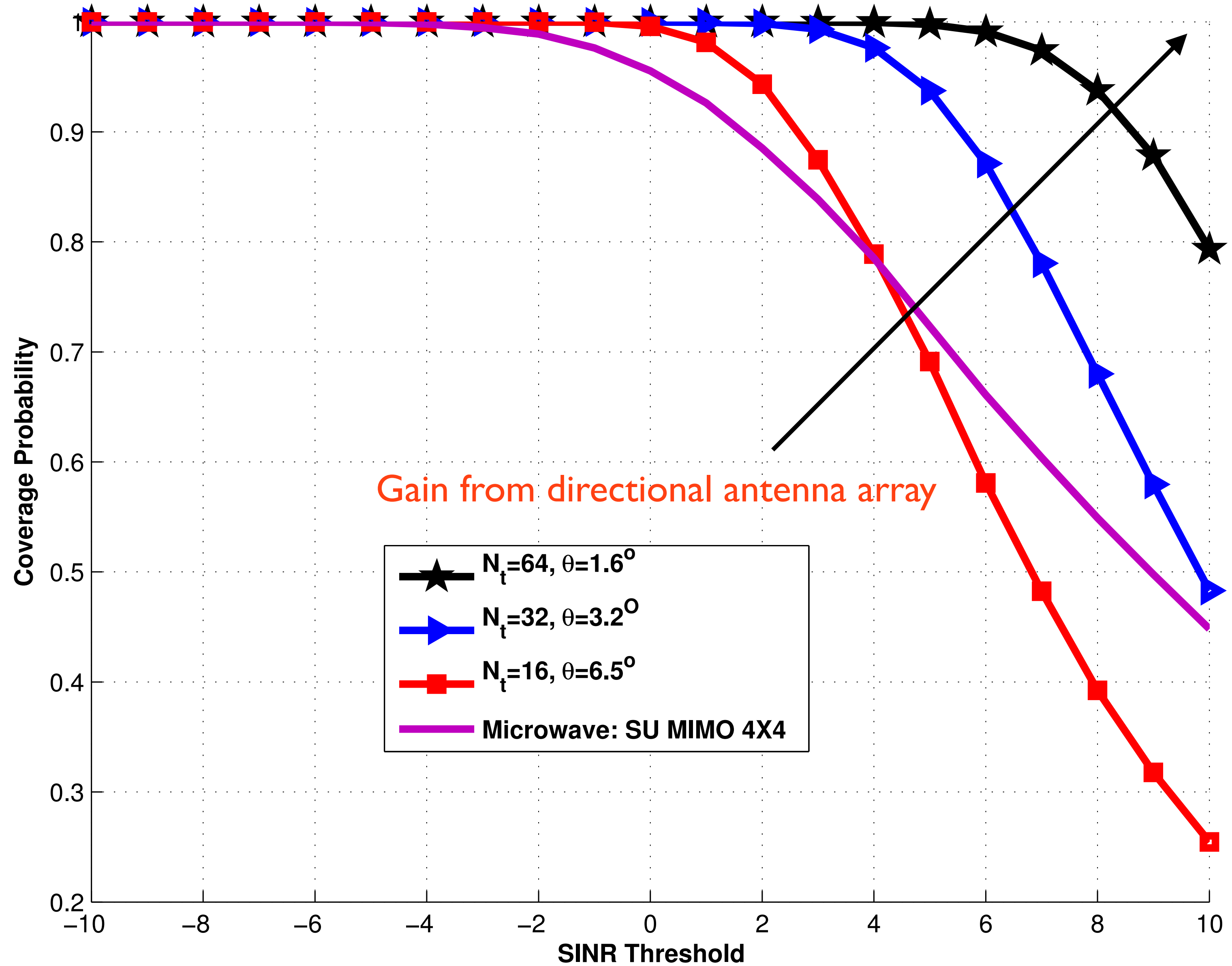
$$\Lambda(x) = 2\pi\lambda\mathbb{E}_h\left[\int_0^{\left(\frac{xh}{K}\right)^{0.25}} t\left(1-\mathrm{e}^{-\beta t}\right)\mathrm{d}t + \int_0^{\sqrt{xh}} t\mathrm{e}^{-\beta t}\mathrm{d}t\right],$$

$$f_{L^*}(x) = -\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{-\Lambda(x)}.$$

Transform interference field into 1D space by Displacement Thm

Coverage Gain from Large Arrays

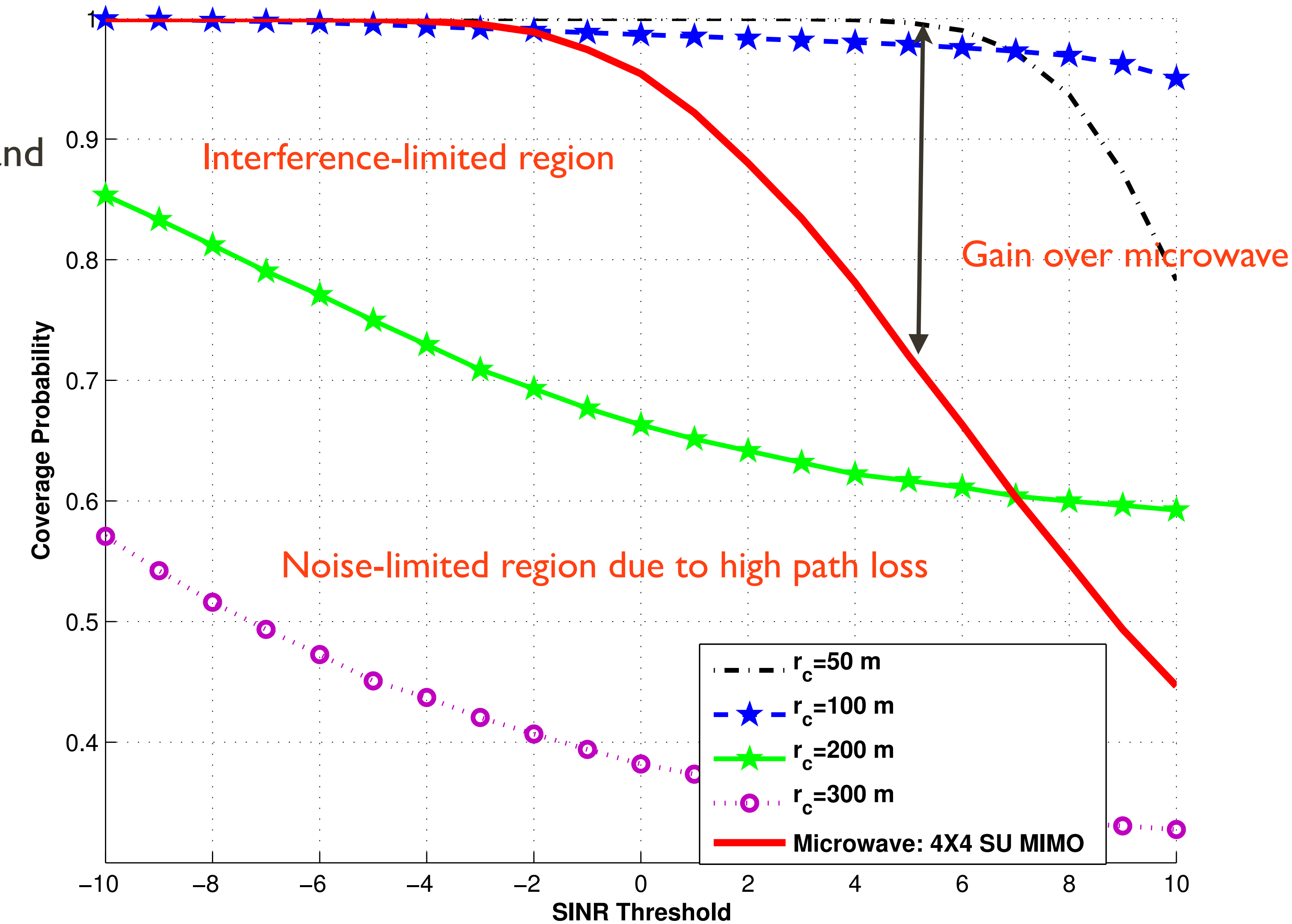
Mobile user 16 antennas
BS density Rc=200 m
Buildings cover 5% of the land
Average building size 15 m by 15 m



- Large arrays provide better coverage probability
  - the more antennas, the smaller beamwidth, the larges array gain
  - Smaller beamwidth provides better coverage
  - mmWave coverage probability comparable to microwave

Coverage Gain from Higher Density

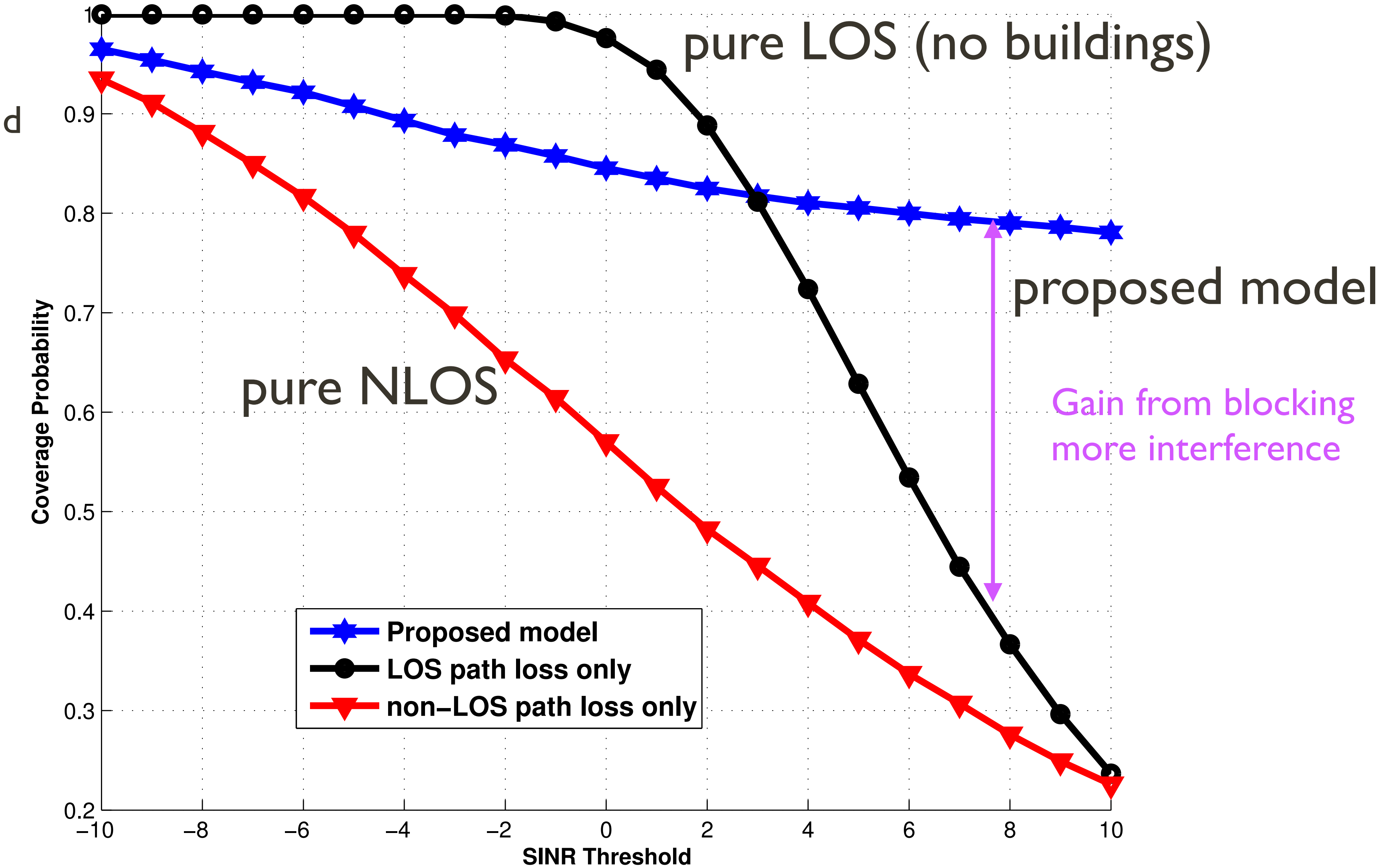
32 antennas at BS
Blockages covers 10% land



- Higher density can also increase coverage probability
  - Coverage probability no longer invariant with BS density
  - Become interference-limited when coverage probability is good

LOS & non-LOS Path Loss

32 antennas at BSs
Blockages covers 10% land
Rc=100 m



- Coverage probability differs in LOS and non-LOS region
  - Need to incorporate blockage model & differentiate LOS and nonLOS
  - Non-LOS coverage probability generally provides a lower bound
  - Buildings may improve coverage by blocking more interference

LOS & Reflection Coverage

Corollary 1 [Coverage probability by LOS BSs]

The coverage probability provided by LOS BSs is

$$\mathbb{P}(SINR > T) = \int_{-\infty}^{\infty}\int_{0}^{\infty} U_1(x,t)U_2(t)f_{L^*}(x)\frac{e^{j2\pi t/T}-1}{j2\pi t}\mathrm{d}x\mathrm{d}t \ ,$$

where

$$U_1(x,s) = \exp\left[-\frac{sx}{M_\mathrm{r}M_\mathrm{t}\rho} + \int_{x}^{\infty}\left(p_\mathrm{t}p_\mathrm{r}\mathrm{e}^{-\frac{sx}{u}} + (1-p_\mathrm{t})p_\mathrm{r}\mathrm{e}^{-\frac{sxm_\mathrm{t}}{uM_\mathrm{t}}} + p_\mathrm{t}(1-p_\mathrm{r})\mathrm{e}^{-\frac{sxm_\mathrm{r}}{uM_\mathrm{r}}} + (1-p_\mathrm{t})(1-p_\mathrm{r})\mathrm{e}^{-\frac{sxm_\mathrm{t}m_\mathrm{r}}{uM_\mathrm{r}M_\mathrm{t}}} - 1\right)\Lambda_1(\mathrm{d}u)\right],$$

$$U_2(s) = \exp\left[\int_{0}^{\infty}\left(p_\mathrm{t}p_\mathrm{r}\mathrm{e}^{-\frac{sx}{u}} + (1-p_\mathrm{t})p_\mathrm{r}\mathrm{e}^{-\frac{sxm_\mathrm{t}}{uM_\mathrm{t}}} + p_\mathrm{t}(1-p_\mathrm{r})\mathrm{e}^{-\frac{sxm_\mathrm{r}}{uM_\mathrm{r}}} + (1-p_\mathrm{t})(1-p_\mathrm{r})\mathrm{e}^{-\frac{sxm_\mathrm{t}m_\mathrm{r}}{uM_\mathrm{r}M_\mathrm{t}}} - 1\right)\Lambda_2(\mathrm{d}u)\right],$$

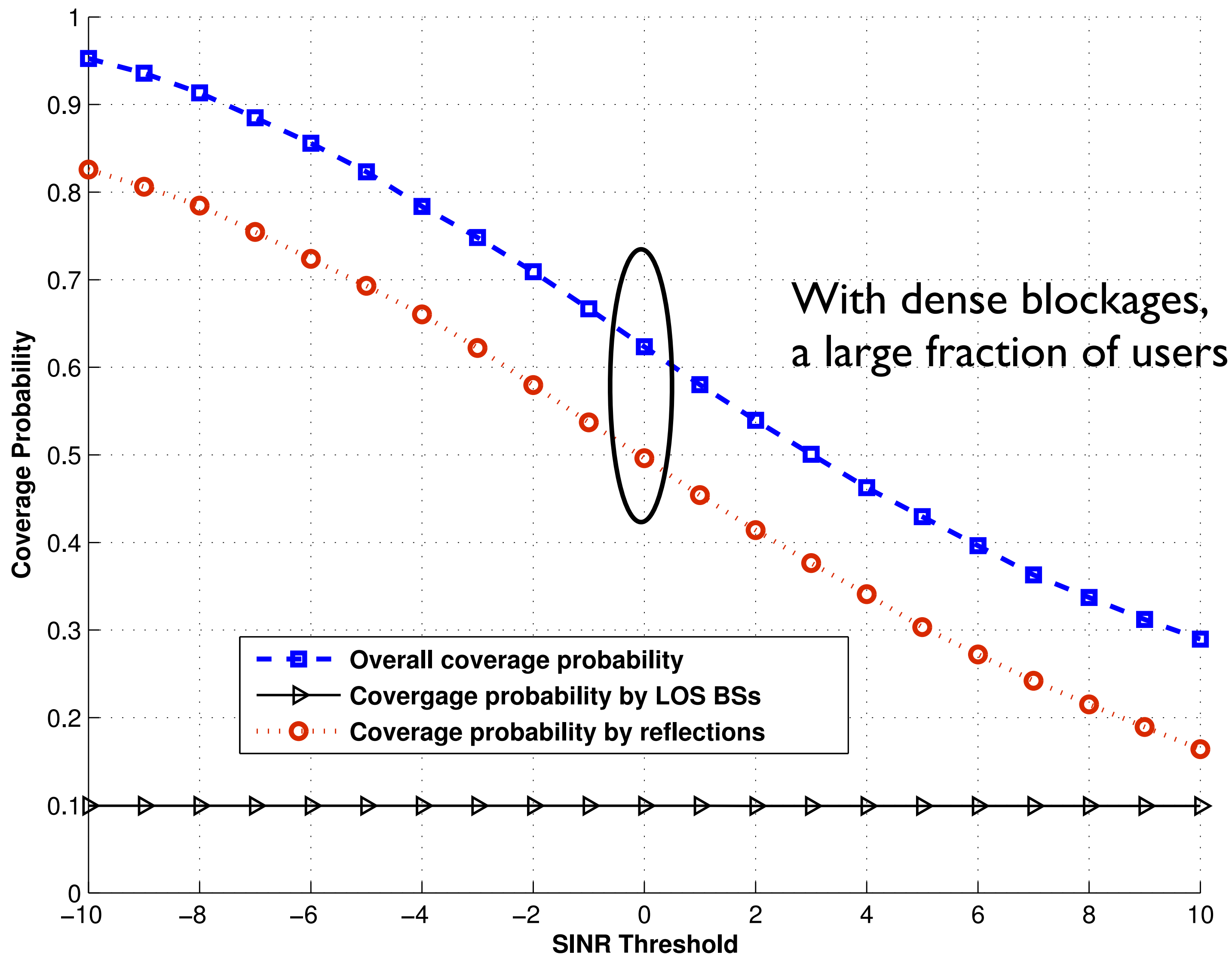
$$\Lambda_1(x) = 2\pi\lambda\int_{0}^{\sqrt{x}} t\mathrm{e}^{-\beta t}\mathrm{d}t, \qquad \Lambda_2(x) = 2\pi\lambda\int_{0}^{\left(\frac{x}{K}\right)^{1/4}} t\left(1-\mathrm{e}^{-\beta t}\right)\mathrm{d}t,$$

$$f_{L^*}(x) = -\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{-\Lambda_1(x)}.$$

Coverage probability by reflections derived in a similar way

Reflections Improve Coverage

128 antennas at BSs
Blockages covers 30% land
(Heavy shadowing case)
Rc=200 m



With dense blockages,
a large fraction of users is non-LOS

- Reflections can establish links in the shadowed areas
  - With dense blockages, most users are served by reflected links
  - Non-LOS links improve the coverage probability of mmWave

Results on Rate

Data Rate Comparison

- Given coverage probability, the achievable rate is

$$\eta = \int_0^\infty \frac{P_c(T)}{1+T} \mathrm{d}T$$

- Microwave network 4X4 MU MIMO with bandwidth 50MHz:
  - Spectrum efficiency is 4.95 bps/ Hz
  - Data rate is 248 Mbps (invariant with the cell size Rc)
- mmWave network with bandwidth 500MHz:

16 antenna at MS
Blockages covers 10% land

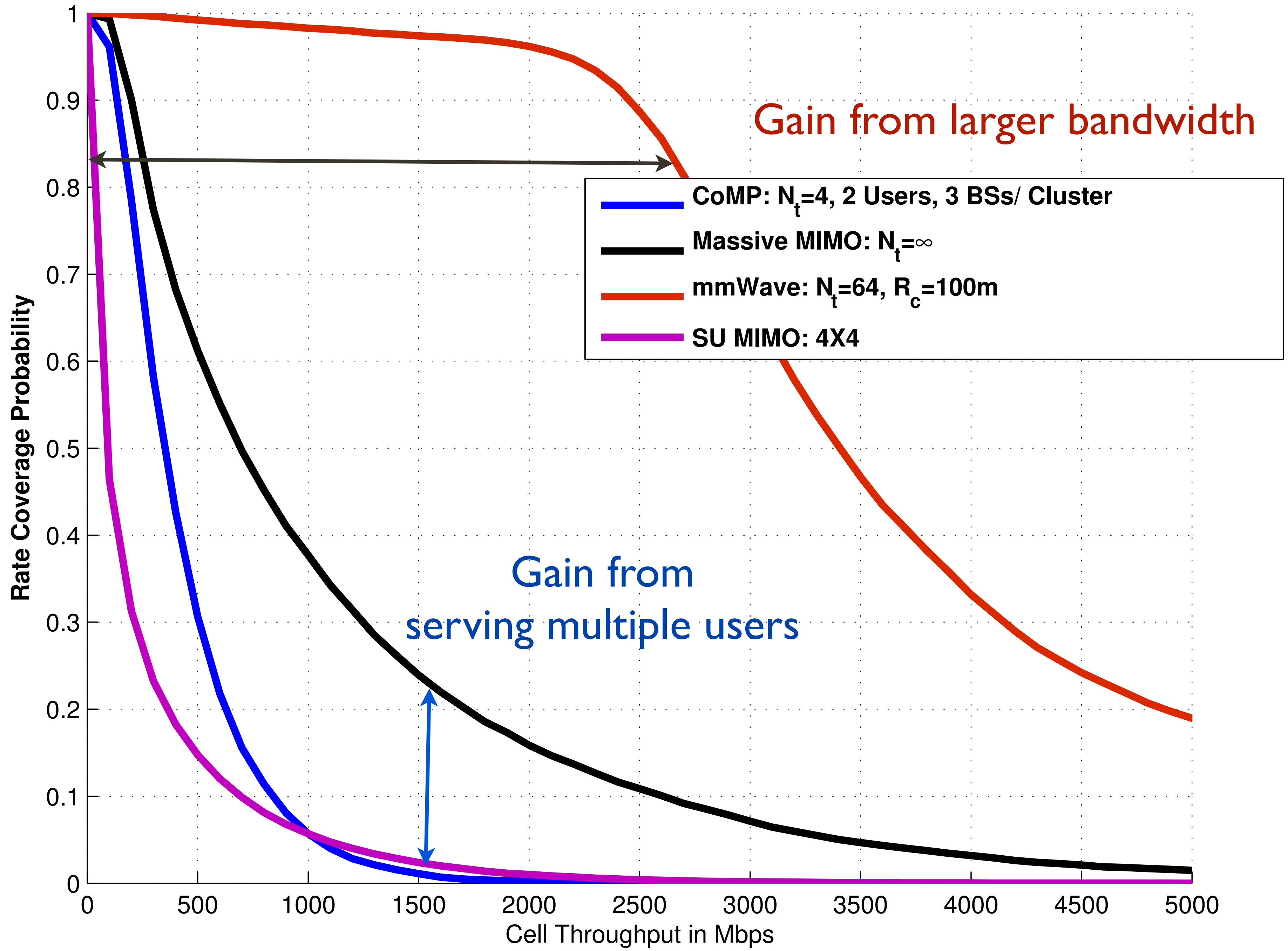
N_t \ R_c	100m	200m
32	3.4Gbps	3.25Gbps
64	3.8Gbps	3.45Gbps

Average cell radius

# of antenna at BS

mmWave achieves high gain in average rate

Cell Throughput Comparison



mmWave can support much higher data rate

* for more information on this setup refer to: Robert W. Heath Jr.," Role of MIMO Beyond LTE: Massive? Coordinated? mmWave?", Workshop on Beyond 3GPP LTE-A ICC 2013.

Conclusions

Going Forward with mmWave

- mmWave coverage probability and rate
  - Need to include both LOS and Non-LOS conditions
  - Interference is reduced by directional antennas and blockages
  - Good rates and coverage can be achieved

- Theoretical challenges abound
  - Analog beamforming algorithms & hybrid beamforming
  - Channel estimation, exploiting sparsity, incorporating robustness
  - Multi-user beamforming algorithms and analysis
  - Microwave-overlaid mmWave system a.k.a. phantom cells
  - More advanced stochastic geometry models including multi-tier

See forthcoming book:

Millimeter Wave Wireless Communication

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Questions?

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